On the Optimal Two Block H^∞ Compensators for Distributed Unstable Plants [1]

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Abstract

In this paper we discuss how the optimal compensator can be calculated in H^∞ mixed sensitivity minimization problems for a class of unstable distributed systems. An operator theoretic (*skew Toeplitz*) approach is taken which builds on a framework developed in the previous papers [12], [13], [14] for calculating the *optimal performance*. We point out that such an optimal controller, which will be infinite dimensional in general, can be approximated using the techniques of [11] to find a corresponding suboptimal finite dimensional compensator.

1 Introduction

In the papers [12], [13], [14], an operator theoretic (*skew Toeplitz*) approach was employed in order to compute the optimal performance for several kinds of two-block H^∞-optimization problems for a broad class of unstable distributed systems. For stable plants, such an approach was first worked out in [17]. An extension of this approach to the unstable case has been presented in [5]. The purpose of this present note is to give an explicit formula for the optimal (distributed) compensator arising in such two-block problems.

In this paper we consider shift-invariant plants which are stabilizable in an L^2 input-output sense, and so possess a coprime factorization over H^∞. We additionally require that the denominator is rational (so the plant has finitely many unstable poles) and we impose a mild smoothness assumption on the numerator outer part. We will see that it is straightforward to compute both the optimal performance and compensator for such plants from a *finite system of linear equations*. Indeed, for the standard mixed sensitivity/complementary sensitivity problem the number of such linear equations only depends on the MacMillan degrees of the weighting filters, and the number of unstable poles of the given plant. Hence, the major restriction of our method is that we require that the unstable distributed plant only have a finite number of unstable poles.

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We would like to mention that the skew Toeplitz techniques employed here have been used to synthesize controllers for several types of flexible structures and delay systems in [9] and [3]. We should also note that besides the frequency domain approach to distributed H^∞ optimization, which we use here to synthesize optimal controllers, there have been other approaches, most notably those based on state space techniques. See [2] for a comprehensive list of references on this approach to distributed H^∞ control.

2 Generalities on Two Block Problems

In this section we will review the reduction of several 2-block H^∞-minimization problems to the computation of the norm of a certain skew Toeplitz operator. We should note that in the finite dimensional case, and stable distributed case this type of reduction is rather standard. However, in the framework of unstable plants given below, there are a few additional technical difficulties. The work here is taken from [13] to which we refer the reader for full details.

We first set up some notation. The Hardy spaces H^2 and H^∞ are defined on the unit disc in the standard way. We denote

$$\begin{aligned} \tilde{H}^\infty &:= \left\{ f \in H^\infty : \overline{f(\bar{z})} = f(z) \right\}, \\ R\tilde{H}^\infty &:= \left\{ \text{rational functions in } \tilde{H}^\infty \right\}. \end{aligned}$$

We consider the feedback configuration of Figure 1 with

$$P = \frac{G_n}{G_d}$$

and $G_n \in \tilde{H}^\infty$, $G_d \in R\tilde{H}^\infty$. We assume that (i) $G_n = m_n G_{no}$, where $m_n \in \tilde{H}^\infty$ is inner (arbitrary) and $G_{no} \in \tilde{H}^\infty$ is outer, and (ii) G_n is analytic and non-zero at the zeros of G_d in the closed unit disc. We also write $G_d = m_d G_{do}$ where $m_d \in R\tilde{H}^\infty$ is inner and $G_{do} \in R\tilde{H}^\infty$ is outer. Under these assumptions there exist $X \in R\tilde{H}^\infty$ and $Y \in \tilde{H}^\infty$ such that the Bezout equation holds:

$$XG_n + YG_d = 1. \tag{1}$$

(To construct solutions of (1), X must be chosen to satisfy a set of interpolation constraints at the zeros of G_d in the closed unit disc so that $Y = (1 - XG_n)/G_d$ belongs to $\tilde{H}^\infty$. Since the constraints are finite in number, X can always be chosen to be rational.) The set of all controllers which stabilize the plant can now be written from the Youla parametrization

$$C = \frac{X + QG_d}{Y - QG_n}$$

for some $Q \in \tilde{H}^\infty$. Let $S := (1+PC)^{-1}$ be the sensitivity function and note that

$$S = 1 - XG_n - QG_nG_d. \tag{2}$$

This form of the sensitivity function will be exploited below.

There are several mixed sensitivity minimization problems which can be reduced to the "standard" 2-block H^∞ problem. For example, consider the feedback configuration of Figure 1, and let $x_1 = 0$. Then in the S and $T = 1 - S$ mixed sensitivity minimization problem we want to minimize the effect of the worst x_2 on the "weighted signals of interest," W_1e_2 and W_2Pe_1, in the energy amplification sense, i.e., we want to find

$$\mu = \inf_{C \text{ stabilizing}} \sup_{x_2 \neq 0} \frac{\left\| \begin{bmatrix} W_1e_2 \\ W_2Pe_1 \end{bmatrix} \right\|_2}{\|x_2\|_2},$$

where $\|\cdot\|_2$ denotes the energy (2-norm). As is well-known, this problem is equivalent to finding

$$\mu = \inf_{\text{stabilizing } C} \left\| \begin{bmatrix} W_1S \\ W_2(S-1) \end{bmatrix} \right\|_\infty$$

where $W_1, W_2 \in R\tilde{H}^\infty$ are given weighting functions with $W_1^{-1}, W_2^{-1} \in R\tilde{H}^\infty$. From (2) we can write

$$\mu = \inf_{Q \in \tilde{H}^\infty} \left\| \begin{bmatrix} W_1 - W_1XG_n \\ -W_2XG_n \end{bmatrix} - \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} QG_nG_d \right\|_\infty$$

Writing $W_1^*W_1 + W_2^*W_2 = G^*G$ where $G, G^{-1} \in R\tilde{H}^\infty$ and performing a unitary transformation we obtain [13]

$$\mu = \inf_{Q \in \tilde{H}^\infty} \left\| \begin{bmatrix} G^{*-1}W_1^*W_1 - XGG_n - QGG_nG_d \\ -W_1W_2G^{-1} \end{bmatrix} \right\|_\infty .$$

Since $G^{*-1}W_1^*W_1 \in RL^\infty$, there exists a finite Blaschke product $b_1 \in R\tilde{H}^\infty$ such that $W_0 := b_1G^{*-1}W_1^*W_1$ and belongs to $R\tilde{H}^\infty$. Thus

$$\mu = \inf_{Q \in \tilde{H}^\infty} \left\| \begin{bmatrix} W_0 - GXb_1G_n - Qb_1GG_nG_d \\ -W_1W_2G^{-1} \end{bmatrix} \right\|_\infty . \tag{3}$$

We now write $Q_1 := GG_{no}Q$, $G_2 := -W_1W_2G^{-1}$. Then under certain mild conditions on the plant and the weights, (3) reduces to

$$\mu = \inf_{Q_1 \in \tilde{H}^\infty} \left\| \begin{bmatrix} W_0 - b_1m_n(GG_{no}X + Q_1G_d) \\ G_2 \end{bmatrix} \right\|_\infty . \tag{4}$$

We refer the reader to [4] for a detailed discussion on this type of step. Now since G_d is rational, we can find a rational $R_1 \in H^\infty$ such that

$$\frac{GG_{no}X - R_1}{G_d} \in \tilde{H}^\infty.$$

Note that R_1 has to satisfy only finitely many interpolation conditions. Defining

$$Q_2 = Q_1 + \frac{GG_{no}X - R_1}{G_d}$$

and $Q_3 = G_{do}Q_2$, we have,

$$\mu = \inf_{Q_3 \in \tilde{H}^\infty} \left\| \begin{bmatrix} W_0 + \hat{W}_0m - m_vQ_3 \\ G_2 \end{bmatrix} \right\|_\infty , \tag{5}$$

where $\hat{W}_0 = -R_1$, $m_v = b_1m_nm_d$, and $m = b_1m_n$. This is a 2-block problem in which W_0, $\hat{W}_0$, G_2 are rational functions in $\tilde{H}^\infty$, m is arbitrary inner (since m_n is arbitrary inner) and $m_v = mm_d$ where m_d is inner rational. Note that the outer part of the plant numerator G_{no} is not assumed to be rational. In Section 4 below, we use the optimal interpolant $Q_{3,opt}$ to find the corresponding optimal controller achieving the performance level μ.

We can do a similar type of reduction for the so-called S and CS mixed sensitivity minimization problem, where again, we want to minimize the effect of worst x_2, but this time the signals of interest are W_1e_2 and W_2e_1. See Figure 1. This leads to the problem of finding

$$\mu = \inf_{\text{stabilizing } C} \left\| \begin{bmatrix} W_1S \\ W_2CS \end{bmatrix} \right\|_\infty \tag{6}$$

where $W_1, W_2 \in R\tilde{H}^\infty$ are given weighting functions with $W_1^{-1}, W_2^{-1} \in R\tilde{H}^\infty$. It is interesting to remark that the problem of robustness optimization in the gap metric ([7]), and its weighted version ([8]) can be written in the form (6), for certain choices of the weighting functions $W_1, W_2 \in R\tilde{H}^\infty$, providing that G_{do} is invertible [10].

For the reduction of (6) to 2-block form we will make an additional assumption that G_{no} is rational. Since $CS = XG_d + QG_d^2$ we can write

$$\mu = \inf_{Q \in \tilde{H}^\infty} \left\| \begin{bmatrix} W_1 \\ 0 \end{bmatrix} - \begin{bmatrix} W_1G_n \\ -W_2G_d \end{bmatrix} X - \begin{bmatrix} W_1G_n \\ -W_2G_d \end{bmatrix} QG_d \right\|_\infty .$$

Since G_{no} is rational we have that $G_n^*G_n$ is rational and we can write $W_1^*G_n^*G_nW_1 + W_2^*G_d^*G_dW_2 = G^*G$ where $G, G^{-1} \in R\tilde{H}^\infty$. Once again a unitary transformation leads to [13]

$$\mu = \inf_{Q \in \tilde{H}^\infty} \left\| \begin{bmatrix} W_1^*W_1G_n^*G^{*-1} - XG - QGG_d \\ W_1W_2G_dG^{-1} \end{bmatrix} \right\|_\infty . \tag{7}$$

Again there exists a finite Blaschke product $b_1 \in R\tilde{H}^\infty$ such that $W_0 := b_1 m_n W_1^* W_1 G_n^* G^{*-1}$ belongs to $R\tilde{H}^\infty$. Define $\hat{W}_0 := GX$, $m := b_1 m_n$ $m_v := b_1 m_n m_d$ and $G_0 := W_1 W_2 G_d G^{-1}$. Then (7) reduces to

$$\mu = \inf_{Q \in \tilde{H}^\infty} \left\| \begin{bmatrix} W_0 - \hat{W}_0 m - Q m_v G G_{do} \\ G_0 \end{bmatrix} \right\|_\infty .$$

Now let $Q_1 := QGG_{do}$. Then as before, under mild conditions on the plant and the weighting functions, it can be shown that

$$\mu = \inf_{Q_1 \in \tilde{H}^\infty} \left\| \begin{bmatrix} W_0 - \hat{W}_0 m - Q_1 m_v \\ G_0 \end{bmatrix} \right\|_\infty . \tag{8}$$

Once again, in Section 4, we use the optimal interpolant $Q_{1,opt}$ to find the corresponding optimal controller achieving the performance level μ.

We consider now the standard form (8). This is the same general form as (5). Note that in the stable plant case $\hat{W}_0 = 0$ and $m_d = 1$, i.e. $m_v = m$. Let $\mathbf{S} : H^2 \to H^2$ denote the unilateral shift, $H(m_v) := H^2 \ominus m_v H^2$ and let $\mathbf{P}_{H(m_v)}$ be the orthogonal projection onto $H(m_v)$. Then it follows from the Commutant Lifting Theorem that $\mu = \|\mathbf{A}\|$ where $\mathbf{A} : H^2 \to H(m_v) \oplus H^2$ is defined by

$$\mathbf{A} := \begin{bmatrix} \mathbf{P}_{H(m_v)} \left(W_0(\mathbf{S}) - \hat{W}_0(\mathbf{S}) m(\mathbf{S}) \right) \\ G_0(\mathbf{S}) \end{bmatrix} . \tag{9}$$

Note that μ^2 is the largest element of $\sigma(\mathbf{A}^*\mathbf{A})$—the spectrum of $\mathbf{A}^*\mathbf{A}$—which consists of the discrete spectrum $\sigma_d(\mathbf{A}^*\mathbf{A})$ (i.e. singular values of $\mathbf{A}$ with finite multiplicity) and its complement $\sigma_e(\mathbf{A}^*\mathbf{A})$, the essential spectrum. We will assume throughout this note that the norm of $\mathbf{A}$ is achieved at a singular value. In [14], there is a full discussion of the essential spectrum, and when the above assumption is valid. In the next section we will summarize our approach to computing the singular values and vectors of operators of the form (9). This will be a key step in computing the optimal compensator.

3 Norm of 2-Block Operator

In this section we sketch our derivation of a finite rank type formula for the norm (and singular values) of the operator $\mathbf{A}$.

Let the operator $\mathbf{A}$ be defined as in (9) where W_0, $\hat{W}_0$, $G_0 \in R\tilde{H}^\infty$, $m \in \tilde{H}^\infty$ is inner (arbitrary), $m_v = m_d m$ and $m_d \in \tilde{H}^\infty$ is a finite Blaschke product. We wish to find $\rho \geq 0$ and $0 \neq y \in H^2$ such that

$$\left(\mathbf{A}^*\mathbf{A} - \rho^2 \mathbf{I} \right) y = 0. \tag{10}$$

(Note that such a ρ will be a singular value and such a y will be a singular vector of the operator $\mathbf{A}$.)
From (9), this is equivalent to

$$\Big\{ \left(W_0(\mathbf{S})^* - \hat{W}_0(\mathbf{S})^* m(\mathbf{S})^* \right) \mathbf{P}_{H(m_v)} \left(W_0(\mathbf{S}) - \hat{W}_0(\mathbf{S}) m(\mathbf{S}) \right) + G_0(\mathbf{S})^* G_0(\mathbf{S}) - \rho^2 \mathbf{I} \Big\} y = 0. \tag{11}$$

Now write

$$W_0 = \frac{B}{K}, \qquad \hat{W}_0 = \frac{C}{K}, \qquad G_0 = \frac{D}{K}$$

where B, C, D, and K are real polynomials (with $K^{-1} \in H^\infty$). Then (11) holds for some $0 \neq y \in H^2$ if and only if

$$\mathbf{R}x := \Big\{ \left(B(\mathbf{S})^* - C(\mathbf{S})^* m(\mathbf{S})^* \right) \mathbf{P}_{H(m_v)} \left(B(\mathbf{S}) - C(\mathbf{S}) m(\mathbf{S}) \right) + D(\mathbf{S})^* D(\mathbf{S}) - \rho^2 K(\mathbf{S})^* K(\mathbf{S}) \Big\} x = 0 \tag{12}$$

holds for some $0 \neq x \in H^2$. Note that we can express such an x as

$$x = K^{-1} y, \quad y \in H^2.$$

In order to solve (12) for ρ and x, one needs to explicitly compute the action of the operator $\mathbf{R}$ on x. This is done by studying the action of $\mathbf{R}$ on the canonical decomposition

$$x = u + m_v v, \;\; u \in H(m_v), \;\; v \in H^2. \tag{13}$$

(See [14] for the details.) What we will need for our present purposes is the following summary of formulae from [14].

First note that since $\mathbf{S}^*\mathbf{S} = \mathbf{I}$ and all polynomials have real coefficients we can write

$$\begin{aligned} &\left\{ D(\mathbf{S})^* D(\mathbf{S}) - \rho^2 K(\mathbf{S})^* K(\mathbf{S}) + B(\mathbf{S})^* B(\mathbf{S}) \right\} \\ &\quad =: \; P(\mathbf{S}, \mathbf{S}^*) \\ &\quad =: \; P_{-n} \mathbf{S}^{*n} + \cdots + P_0 + \ldots + P_n \mathbf{S}^n \end{aligned}$$

where in fact $P_i = P_{-i}$. Similarly we can write

$$\begin{aligned} &\left\{ D(\mathbf{S})^* D(\mathbf{S}) - \rho^2 K(\mathbf{S})^* K(\mathbf{S}) \right\} \\ &\quad =: \; Q(\mathbf{S}, \mathbf{S}^*) \\ &\quad =: \; Q_{-n} \mathbf{S}^{*n} + \cdots + Q_0 + \ldots + Q_n \mathbf{S}^n \end{aligned}$$

where $Q_i = Q_{-i}$.

In particular, one can show that for the condition $\mathbf{R}x = 0$, to hold, we need the following two conditions:

$$0 \;=\; Q(z, z^{-1}) v + T_v(z) \Phi, \tag{14}$$

$$0 \;=\; P(z, z^{-1}) u + T_u(z) \Phi, \tag{15}$$

where Φ is constant complex vector of length $3n + 2\ell$, and the vector-valued functions T_v and T_u can be explicitly computed form the given problem data as in [14].

These two equations can be utilized to write down an explicit system of $3n + 2\ell$ linear equations in $3n + 2\ell$ unknowns (the entries of Φ) which determines the necessary and sufficient conditions for the existence of $u \in H(m_v)$ and $v \in H^2$ (not both zero) satisfying $\mathbf{R}(u + m_v v) = 0$, from which we derive the required singular vector via

$$y := K(u + m_v v).$$

These equations can be expressed in matricial form as

$$\Theta(\rho) \Phi = 0,$$

where $\Theta(\rho)$ is an explicitly computable $(3n + 2\ell) \times (3n + 2\ell)$ matrix. (See [14] for the formula for Θ.) Note that the optimal performance μ can be characterized as the largest ρ for which $\sigma_{min}(\Theta(\rho)) = 0$.

We should finally note that Handong Tu and Kathryn Lenz of the University of Minnesota at Duluth have coded the above procedure into a MATLAB program.

4 Optimal Compensators

In this section, we use our knowledge of the singular vectors and singular values of $\mathbf{A}$ to write down the optimal compensator. This is done in terms of the equation

$$\Theta(\mu)\Phi_o = 0, \tag{16}$$

where the optimal performance μ and the corrresponding optimal Φ_o are computed as above. Then using the theory sketched in the previous section, we may construct from equations (15) and (14), $u \in H(m_v)$, $v \in H^2$, and

$$y_o(z) := K(z)(u(z) + m_v(z)v(z)),$$

such that

$$\mathbf{A}^*\mathbf{A}y_o = \mu^2 y_o,$$

where

$$\mathbf{A} := \begin{bmatrix} \mathbf{P}_{H(m_v)}(W_o(\mathbf{S}) - \hat{W}_0(\mathbf{S})m(\mathbf{S})) \\ G_0(\mathbf{S}) \end{bmatrix}.$$

Now it is easy to find the corresponding Schmidt pair. Indeed, setting

$$\psi := \mathbf{A}y_o,$$

we see that

$$\mathbf{A}^*\psi = \mu^2 y_o.$$

Define

$$\hat{\mathbf{A}} := \frac{\mathbf{A}}{\mu}, \qquad \hat{\psi} := \frac{\psi}{\mu}.$$

Then

$$\hat{\mathbf{A}}^*\hat{\psi} = y_o, \qquad \hat{\mathbf{A}}y_o = \hat{\psi}.$$

Note that $\|\hat{\mathbf{A}}\| = 1$. From [15], we have

$$\hat{\mathbf{A}}y_o := \frac{1}{\mu}\begin{bmatrix} \mathbf{P}_{H(m_v)}(W_o - \hat{W}_0 m)y_o \\ G_0 y_o \end{bmatrix} = \frac{1}{\mu}\begin{bmatrix} W_o - \hat{W}_0 m - m_v q_{opt} \\ G_0 \end{bmatrix} y_o$$

for some $q_{opt} \in H^\infty$. (q_{opt} is called the *optimal interpolant* from which we will now derive the optimal compensator.)

Clearly, we have

$$\mathbf{P}_{H(m_v)}(W_o - \hat{W}_0 m)y_o = (W_o - \hat{W}_0 m - m_v q_{opt})y_o. \tag{17}$$

Further,

$$\mathbf{P}_{H(m_v)}(W_o - \hat{W}_0 m)y_o = (W_o - \hat{W}_0 m)y_o - m_v \mathbf{P}_{H^2}\bar{m}_v(W_o - \hat{W}_0 m)y_o. \tag{18}$$

Hence, from (17) and (18), we have

$$q_{opt} = \frac{\mathbf{P}_{H^2}\bar{m}_v(W_o - \hat{W}_0 m)y_o}{y_o}.$$

Recall that

$$W_0 = B/K, \ \ \hat{W}_0 = C/K, \ \ y_o = (u + m_v v)K.$$

Thus, from the above, we see that

$$q_{opt} = \frac{\mathbf{P}_{H^2}B(\bar{m}_v u) - \mathbf{P}_{H^2}C\bar{m}_d u + (B - Cm)v}{K(u + m_v v)}. \tag{19}$$

This can be written as,

$$q_{opt}(z) = K(z)^{-1}\frac{T_b(z)\Phi_o + (B(z) - C(z)m(z))Q(z,z^{-1})^{-1}T_v(z)\Phi_o}{P(z,z^{-1})^{-1}T_u(z)\Phi_o + m_v(z)Q(z,z^{-1})^{-1}T_v(z)\Phi_o}, \tag{20}$$

where $P(z, z^{-1})$ and $Q(z, z^{-1})$ are as defined above. The functions $T_b(z)$, $T_u(z)$, and $T_v(z)$ can explicitly computed from the problem data B, C, D, K, m, m_v. The formulae for $T_u(z)$ and $T_v(z)$ are given in [14]. According to the way Φ is defined in [14], the function $T_b(z)$ takes the form:

$$T_b(z) = [1 \ \ z \ \ \ldots \ \ z^{n-1}]\begin{bmatrix} B_n & \cdots & B_1 & 0 & 0 & \cdots \\ 0 & \ddots & \vdots & 0 & 0 & \cdots \\ 0 & 0 & B_n & 0 & 0 & \cdots \end{bmatrix},$$

where the B_i's are the coefficients of $B(z)$, i.e. $B_0 + B_1 z^1 + \cdots + B_n z^n := B(z)$.

Using q_{opt} we can compute the optimal controller for the mixed sensitivity problems discussed in Section 2.

Case 1: S and T Problem.

Note form Section 2 that the optimal controller is given by

$$C_{opt} = \frac{X + G_d Q_{opt}}{Y - G_n Q_{opt}}$$

where

$$Y = \frac{1 - G_n X}{G_d}.$$

In terms of the functions R_1, G, Q_1, Q_2, Q_3 introduced in Section 2, we have that

$$Q_{opt} = \frac{G^{-1}G_{no}^{-1}(m_d Q_{3,opt} + R_1) - X}{G_d}$$

where $Q_{3,opt} = q_{opt}$ as defined in equation (20). This leads us to

$$C_{opt} = G_d G_{no}^{-1} G^{-1}\frac{(m_d Q_{3,opt} + R_1)}{1 - m_n G^{-1}(m_d Q_{3,opt} + R_1)}.$$

Remark: The Bezout identity and the definition of R_1 imply that

$$1 - m_n G^{-1}(m_d Q_{3,opt} + R_1)$$

becomes zero at the zeros of m_d. Therefore, the G_d term in the controller does not cancel the open right half plane poles of the plant. However, the "optimal" controller tries to invert the outer part of the plant, which means that it will cancel the imaginary axis poles and zeros, and it may be improper. These problems can be removed by a judicious choice of the weighting functions in the original problem definition.

Case 2: S and CS problem.

In this case, the optimal controller is given by

$$C_{opt} = G_d \frac{(X + G^{-1} m_d Q_{1,opt})}{1 - G_n(X + G^{-1} m_d Q_{1,opt})},$$

where $Q_{1,opt} := q_{opt}$.

Remark: Similarly to the above case, the G_d term in the controller does not cancel the open right half plane poles of the plant. However, it cancels the imaginary axis poles. Generically, the optimal controller for this case is proper, and does not cancel the imaginary axis zeros of the plant.

As far as the computation of the optimal controller is concerned, the key in both cases is to find q_{opt} from μ and Φ_o and substitute it into the C_{opt} expressions given above. The optimal interpolant q_{opt} can be explicitly computed from the problem data, as shown in this note. Finally, we point out that such an optimal controller, which will be infinite dimensional in general, can be approximated using the techniques of [11] to find a corresponding suboptimal finite dimensional compensator.

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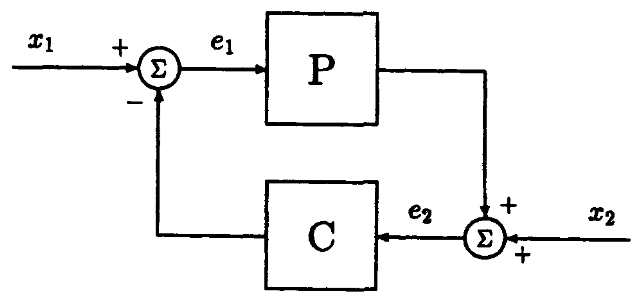

Figure 1: Standard feedback configuration